Aligned natural inflation in the Large Volume Scenario

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Introduction

Axion physics provides a powerful testing ground for string theory.

- String compactifications contain many axions: for example, from fluxes on compact cycles.
- Axions are promising candidates for inflation, since the shift symmetry protects the potential against quantum corrections.
- However, this typically requires trans-Planckian field excursions.
- Models with multiple axions are constrained by the weak gravity conjecture (WGC) [see Gary Shiu's talk for an overview].

These qualitative statements invite the following questions.

1. Can we realise a full, explicit string embedding of axion inflation?
2. Can this help us to quantify constraints such as the WGC?

Aligned natural inflation

Let us review (and rephrase) the aligned natural inflation scenario. [Kim, Nilles, Peloso (2004)]

Consider a two-axion model with a periodic potential,

$$V = \Lambda_1^4 \left[1 - \cos\left(\frac{Q_{11}}{f_1}\phi_1 + \frac{Q_{12}}{f_2}\phi_2\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{Q_{21}}{f_1}\phi_1 + \frac{Q_{22}}{f_2}\phi_2\right)\right] ,$$

where ϕ_1 and ϕ_2 are canonically normalized.

In the lattice basis (where the axions have periodicity 2π), the kinetic matrix is the Kähler metric, $K_{ij} = \mathrm{diag}(f_1^2, f_2^2)/2$.

Note: if K_{ij} is non-diagonal, we can simply diagonalize it and the same arguments follow.

$Q_{Ij} \in \mathbb{Z}$ are instanton charges: in string theory they represent e.g. winding numbers of D-branes on various cycles.

Aligned natural inflation: change of basis

Change basis via an $SO(2)$ rotation,

$$\begin{pmatrix} \phi_\xi \\ \phi_\psi \end{pmatrix} = \frac{1}{\sqrt{f_1^2 Q_{12}^2 + f_2^2 Q_{11}^2}} \begin{pmatrix} f_1 Q_{12} & -f_2 Q_{11} \\ f_2 Q_{11} & f_1 Q_{12} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} .$$

Then the potential can be written as

$$V = \Lambda_1^4 \left[1 - \cos\left(\frac{Q_{1\psi}}{f_\psi}\phi_\psi\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{Q_\xi}{f_\xi}\phi_\xi + \frac{Q_{2\psi}}{f_\psi}\phi_\psi\right)\right],$$

where we have defined decay constants in the new basis,

$$f_\psi = \frac{f_1 f_2 \sqrt{Q_{11}^2 + Q_{12}^2}}{\sqrt{f_1^2 Q_{12}^2 + f_2^2 Q_{11}^2}}, \qquad f_\xi = \frac{\sqrt{f_1^2 Q_{12}^2 + f_2^2 Q_{11}^2}}{\sqrt{Q_{11}^2 + Q_{12}^2}} .$$

Here there is no special hierarchy between $\{f_\psi, f_\xi\}$ and $\{f_1, f_2\}$.

Charge alignment

We have also defined charges in the new basis,

$$Q_{1\psi} = \sqrt{Q_{11}^2 + Q_{12}^2}\,,$$
$$Q_{2\psi} = \left(\frac{f_1^2 Q_{12} Q_{22} + f_2^2 Q_{11} Q_{21}}{f_1^2 Q_{12}^2 + f_2^2 Q_{11}^2} \right) Q_{1\psi}\,;$$

$$Q_\xi = \frac{Q_{21} Q_{12} - Q_{11} Q_{22}}{\sqrt{Q_{11}^2 + Q_{12}^2}}\,.$$

Again, there is no special hierarchy. However, in the alignment limit, $Q_{11} Q_{22} \simeq Q_{21} Q_{12}$, we have $|Q_\xi| \ll 1$ and ϕ_ξ becomes light. Integrating out ϕ_ψ gives the single-axion effective potential,

$$V' \simeq \Lambda_2^4 \left[1 - \cos\left(\frac{\phi_\xi}{f_{\rm eff}} \right) \right]\,, \qquad f_{\rm eff} \equiv \frac{f_\xi}{Q_\xi}\,.$$

Thus, for $|Q_\xi| \ll 1$ we may have $f_{\rm eff} > M_{\rm Pl}$, even while $f_\xi < M_{\rm Pl}$.

Weak gravity conjecture

It is expected that effective field theories that are consistent with quantum gravity should satisfy the weak gravity conjecture (WGC).

[Arkani-Hamed, Motl, Nicolis, Vafa (2006)]

- A $U(1)$ gauge theory with coupling g should contain a particle of charge q and mass m, such that

$$\frac{qgM_{\rm Pl}}{m} \geq \left(\frac{\mathcal{Q}M_{\rm Pl}}{M}\right)_{\text{extremal BH}} = \sqrt{\frac{1}{2}+\frac{\alpha^2}{2}}\,,$$

  where we included a dilaton coupling via $e^{-\alpha\phi}F_{\mu\nu}F^{\mu\nu}$.
- Generalise to p-forms and $(p-1)$-dimensional charged objects.
- What about $p=0$? By analogy, an axion with decay constant f requires an instanton with “charge” Q and action S, such that

$$z \equiv \frac{QM_{\rm Pl}}{fS} \geq \left(\frac{\mathcal{Q}M_{\rm Pl}}{S}\right)_{\text{extremal ??}}\,.$$

Weak gravity conjecture (for axions)

What is the corresponding extremal object? Euclidean wormholes. [Giddings, Strominger (1988)]

Including dilaton dependence [Hebecker, Mangat, Theisen, Witkowski (2016)] [Andriolo, Huang, Noumi, Ooguri, Shiu (2020)]

$$\left(\frac{\mathcal{Q}M_{\rm Pl}}{S}\right)_{\rm WH} = \frac{\alpha}{2\sin\left(\frac{\pi\alpha}{4\sqrt{2/3}}\right)} \geq \sqrt{\frac{2}{3}}\,.$$

Solution only exists for $\alpha < 2\sqrt{2/3}$ [c.f. Pablo Soler's talk, $\alpha \to \beta$].

For a Kähler modulus $T = \tau + ib$ with Kähler potential $K = -2\ln(\tau^p)$,

$$\mathcal{L} = \frac{1}{2}M_{\rm Pl}^2\left(R + p\frac{(\partial_\mu\tau)^2}{\tau^2} + p\frac{(\partial_\mu b)^2}{\tau^2}\right) \supset \frac{1}{2}(2\pi f_0)^2 e^{-\alpha\tilde{\phi}/M_{\rm Pl}}(\partial_\mu b)^2\,,$$

after normalisation, with $\alpha = 2\sqrt{1/p}$ (e.g. bulk volume: $p = 3/2$).

Multiple axions/instantons: convex hull of charge-to-action ratios $\mathbf{z}_I$ should contain the ball of radius $r_{\rm CHC} \equiv \sqrt{2/3}$.

The Large Volume Scenario

We now try to embed aligned natural inflation into string theory.

For parametric control, consider the Large Volume Scenario (LVS).
[Balasubramanian, Berglund, Conlon, Quevedo (2005)]

- Type IIB CY3 orientifold, fluxes stabilising dilaton and CS moduli.
  [Giddings, Kachru, Polchinski (2001)]; [Gukov, Vafa, Witten (1999)]
- CY3 is fibered (K3 over $\mathbb{CP}^1$) and contains small internal cycles $\Rightarrow$ anisotropic bulk, swiss-cheese geometry.
- Benchmark example: $\mathbb{CP}^4_{[1,1,2,2,6]}(12)$ with additional small cycle,
  [Cicoli, Conlon, Quevedo (2008)]

$$\mathcal{V} = \alpha(\tau_1^{1/2}\tau_2 - \gamma\tau_s^{3/2})\,.$$

- α' corrections to K + gaugino condensation of D7-brane stack on small cycle giving non-pert. $W \Rightarrow$ stabilise $\mathcal{V}$ and τ_s (and b_s).

LVS: anisotropic bulk, loop corrections

So far, only small cycles and overall volume are stabilised.

- Stabilise fibre modulus τ_1 using string loop corrections.
  [Berg, Haack, Körs (2005)]; [Cicoli, Conlon, Quevedo (2008)]
- Axions b_1 and b_2 stabilised by D7-brane stacks on bulk cycles.
  Leading-order potential:

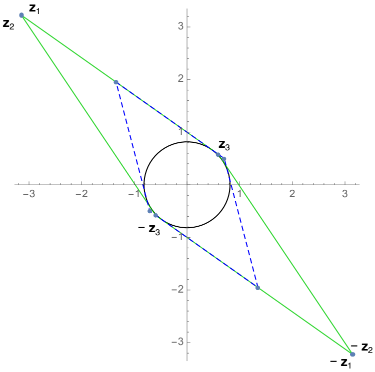
$$V = V_0 - \sum_{I=1}^{3} \lambda_I^4 e^{-S_I} \cos(c_I Q_{Ij} b_j)\,, \qquad S_I = c_I Q_{Ij} \tau_j .$$

- The charges Q_{Ij} are the no. of times D7 stack I wraps cycle j.
  Note: 3 instantons required, to satisfy WGC.
- This potential can realise aligned natural inflation.

Caveat: We do not explicitly realise a de Sitter vacuum, but assume uplifting to V_0 (e.g. from anti-D3 branes) such that the minimum is dS.

Constraints: WGC and Kähler cone condition

- To ensure positive volumes, all τ_i should lie in the Kähler cone.
- For the example $\mathbb{CP}^4_{[1,1,2,2,6]}(12)$, this imposes $\tau_2 > 4\tau_1/3$.
- The instanton "charge-to-mass" vectors $\mathbf{z}_1$ and $\mathbf{z}_2$ should be aligned; to satisfy the WGC, $\mathbf{z}_3$ should be orthogonal to them.
- WGC imposes an upper bound on τ_2/τ_1 from charge ratios.



Overall, preference for almost isotropic geometry, $\tau_2 \sim \tau_1$.

Alignment and suppression of S_3 potential possible if we choose

$$Q_{11} < 0\,, \quad Q_{12} > 0\,, \quad Q_{21} < 0\,, \quad Q_{22} > 0\,, \quad Q_{31} > 0, \quad Q_{32} > 0\,.$$

Hierarchies and observational constraints

The extended no-scale structure and large volume in LVS ensure scale separation and control of corrections:

- leading LVS minimum at $\mathcal{O}(\mathcal{V}^{-3})$;
- string loop corrections at $\mathcal{O}(\mathcal{V}^{10/3} g_s^2)$ (controlled for small g_s);
- bulk axion potential at $\mathcal{O}\left(e^{-k\mathcal{V}^{2/3}}\right)$ for some $k \sim \mathcal{O}(1)$.

To match observations, the height of the inflationary potential should match the amplitude of the observed scalar power spectrum. This requires a volume

$$\mathcal{V} \sim \mathcal{O}(10^{1\text{–}2})$$

$\Rightarrow$ the "not-so-Large Volume Scenario".

Parameters and observables

Parametrise alignment using a single charge Q, setting

$$\begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \\ Q_{31} & Q_{32} \end{pmatrix} = \begin{pmatrix} -Q & Q+1 \\ -Q+1 & Q \\ Q+1 & Q \end{pmatrix} \quad \Rightarrow \quad Q_\xi = \frac{1}{\sqrt{2Q^2+2Q+1}} \, .$$

Recall that $f_{\text{eff}} = f_\xi / Q_\xi$, so increasing Q enhances f_{eff}.

Consider benchmark parameters for three cases:

	g_s	N_s	τ_s	$\mathcal{V}$	τ_2/τ_1	τ_1	τ_2	N_1	N_2
Case 1	0.1	7	4.336	33.86	$\sqrt{2}$	13.187	17.649	18	16
Case 2	0.1	10	4.577	16.186	$\sqrt{2}$	8.06	11.40	16	14
Case 3	0.05	20	9.1547	45.781	$4/3$	16.77	22.36	22	20

Here N_I is the number of D7 branes in the Ith stack, i.e. the rank of the condensing gauge group. Also, set $\alpha = \frac{1}{2}, \gamma = 1, e^{K_0} = W_0 = A_s = 1$.

Mass hierarchy

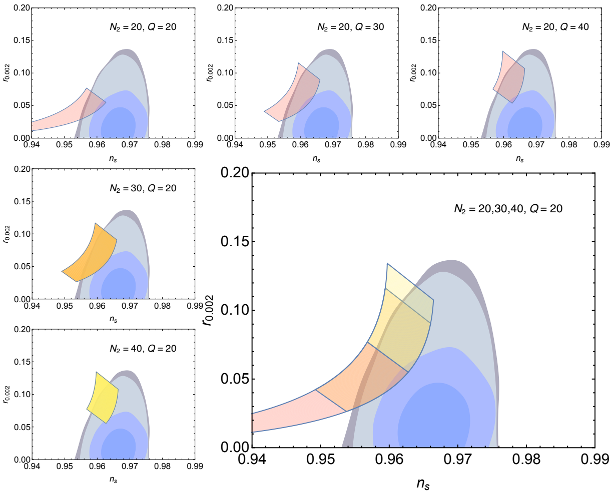
Mass scale	Case 1 (GeV)	Case 2 (GeV)	Case 3 (GeV)
$2\pi M_s = \frac{g_s M_{\rm Pl}}{\sqrt{4\pi\mathcal{V}}}$	7.3×10^{16}	1.1×10^{17}	3.1×10^{16}
$M_{\rm KK} \sim \frac{2\pi M_s}{\tau_2^{1/4}}$	3.5×10^{16}	5.7×10^{16}	1.4×10^{16}
$M_{T_s} \sim \frac{M_s}{\mathcal{V}^{1/2}}$	2.0×10^{15}	4.2×10^{15}	7.4×10^{14}
$M_{S,U} \sim \frac{g_s M_s}{\mathcal{V}^{1/2}}$	4.0×10^{14}	8.4×10^{14}	7.4×10^{13}
$M_{\mathcal{V}} \sim \frac{g_s M_s}{\mathcal{V}}$	3.4×10^{13}	1.0×10^{14}	5.5×10^{12}
$M_\perp \sim \frac{g_s M_s}{\mathcal{V}\tau_1^{1/4}}$	1.8×10^{13}	6.2×10^{13}	2.7×10^{12}
$m_\psi = \frac{Q_{1\psi}\Lambda_1^2}{f_\psi}$	4.4×10^{14}	4.3×10^{15}	5.9×10^{11}
$m_\xi = \frac{\Lambda_2^2}{f_{\rm eff}}$	1.8×10^{12}	8.1×10^{12}	2.8×10^{8}
$H_{\rm inf} \sim \frac{\Lambda_2^2}{\sqrt{3}M_{\rm Pl}}$	2.0×10^{12}	1.6×10^{13}	6.5×10^{8}

Estimated mass scales for the three benchmark cases, with parametric scaling [Conlon, Quevedo, Surulitz (2005); Cicoli, Mazumdar (2010)].

We have used $Q = 9$ (Case 1), $Q = 12$ (Case 2) and $Q = 20$ (Case 3).

Note: larger $Q \Rightarrow$ further suppression.

Predictions for inflation



Plots of the spectral index n_s and the tensor-to-scalar ratio $r_{0.002}$. We take $\tau_2 = \sqrt{2}\tau_1$ and the range $10 \leq \tau_1 \leq 20$ and $50 \leq N_e \leq 60$.

Summary

- Aligned natural inflation allows a trans-Planckian field excursion from an alignment of instanton charges Q, while the underlying decay constants remain sub-Planckian.
- Both Euclidean wormholes and normalisation of Kähler moduli from string compactifications suggest that the $\mathcal{O}(1)$ constant in the weak gravity conjecture for axions is $\sqrt{2/3}$.
- We embedded aligned natural inflation in an explicit string model using the Large Volume Scenario.
- Consistent realisation seems possible in a fibred model with $\mathcal{V} \sim \mathcal{O}(10^{1-2})$, $Q \sim 20$ and almost-isotropic geometry.
- Important issues: visible sector, reheating, dS uplift, electroweak hierarchy, checking consistency...